

Instructions: Try these problems from the textbook as practice (answers below).

Problems

1-6. Evaluate the iterated integral.

$$1. \int_1^5 \int_0^x (8x - 2y) \, dy \, dx$$

$$3. \int_0^1 \int_0^y x e^{y^3} \, dx \, dy$$

$$5. \int_0^1 \int_0^{s^2} \cos(s^3) \, dt \, ds$$

11-14, 23-28. Evaluate the double integral.

$$11. \iint_D \frac{y}{x^2 + 1} \, dA, \, D = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}\}$$

$$13. \iint_D e^{-y^2} \, dA, \, D = \{(x, y) : 0 \leq y \leq 3, 0 \leq x \leq y\}$$

$$23. \iint_D x \cos y \, dA, \, D \text{ is bounded by } y = 0, \, y = x^2, \, x = 1$$

$$25. \iint_D y^2 \, dA, \, D \text{ is the triangular region with vertices } (0, 1), (1, 2), (4, 1)$$

$$27. \iint_D (2x - y) \, dA, \, D \text{ is bounded by the circle with center the origin and radius 2}$$

17. Evaluate $\iint_D x \, dA$ in two ways by expressing the region D enclosed by $y = x$, $y = 0$, $x = 1$ as both

$$D = \{(x, y) : a \leq x \leq b, f(x) \leq y \leq g(x)\} \quad \text{and} \quad D = \{(x, y) : c \leq y \leq d, p(y) \leq x \leq q(y)\}.$$

31-40. Find the volume of the given solid.

31. Under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

33. Under the surface $z = xy$ and above the triangle with vertices $(1, 1)$, $(4, 1)$, and $(1, 2)$.

35. The tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

37. Enclosed by the cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$, $y = 4$.

39. Bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y = z$, $x = 0$, $z = 0$ in the first octant.

43-46. Find the volume of the solid by subtracting two volumes.

43. The solid enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$, $2x + 2y - z + 10 = 0$.

45. The solid under the plane $z = 3$, above the plane $z = y$, and between the parabolic cylinders $y = x^2$ and $y = 1 - x^2$.

55-60. Sketch the region of integration and change the order of integration.

$$55. \int_0^1 \int_0^y f(x, y) \, dx \, dy$$

$$57. \int_0^{\pi/2} \int_{\sin x}^1 f(x, y) \, dy \, dx$$

$$59. \int_1^2 \int_0^{\ln x} f(x, y) \, dy \, dx$$

61-66. Evaluate the integral by reversing the order of integration.

$$61. \int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy$$

$$63. \int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} \, dy \, dx$$

$$65. \int_0^1 \int_{\arcsin y}^{\pi/2} e^{x^4} \, dx \, dy$$

71. Find the average of $f(x, y) = xy$ over the region D which is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.

Answers

1. $\frac{868}{3}$

3. $\frac{1}{6}(e-1)$

5. $\frac{1}{3}\sin(1)$

11. $\frac{1}{4}\ln(17)$

13. $\frac{1}{2}(1-e^{-9})$

17. Type I: $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x\}$.

Type II: $D = \{(x, y) : 0 \leq y \leq 1, y \leq x \leq 1\}$.

$\frac{1}{3}$.

23. $\frac{1}{2}(1-\cos(1))$

25. $\frac{11}{3}$

27. 0

31. $\frac{3}{4}$

33. $\frac{31}{8}$

35. $\frac{16}{3}$

37. $\frac{128}{15}$

39. $\frac{1}{3}$

43. $\frac{64}{3}$

45. $\frac{10}{3\sqrt{2}}$ or $\frac{5\sqrt{2}}{3}$

61. $\frac{1}{6}(e^9-1)$

63. $\frac{2}{9}(2\sqrt{2}-1)$

65. $\frac{1}{3}(2\sqrt{2}-1)$

71. $\frac{3}{4}$